

# Unsteady Aerodynamic Model for Thin Wings with Evolutive Vortex Sheets

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## ABSTRACT :

*This paper presents recent advances in a computing method for unsteady inviscid incompressible flows around thin lifting surfaces with arbitrary motion. This involves the velocity field, deriving from a potential, the pressure field and the free vortex sheet characteristics (geometry and vortex strength). When considering high angles of attack, the separation lines along which the vortex sheets are shed are the trailing edge, as well as wing tips or parts of the leading edge. Compared to classical methods of singularity, such as panel methods, an improved vortex wake model was constructed. It consists in representing the wake by a vortex particle concept, particularly well suited for the prediction of its unsteady deformations. This approach stems from the unsteady vortex sheet theory established by MUDRY, in which the vortex sheet is considered as a median layer, on which is founded the concept of continuous vortex particle. This ensures the computing method to compare favorably with the classical methods in terms of flexibility and computing costs. Computed unsteady flow-field simulations are presented for several cases and compared with some available experimental data.*

## 1 INTRODUCTION

In order to develop computational methods to analyse complex aircraft flight conditions, the simulation of unsteady aerodynamics and the resulting wake dynamics is necessary. It is much needed in aeronautical disciplines such as flight dynamics and flight simulation and for time-dependent structural load analysis. Detailed solution of the complete nonlinear fluid dynamic equation along time-dependent flight path is still costly, particularly because of the computational grid to encompass large wake histories along complex arbitrary path. An alternative approach is to use simplified fluid dynamic equations while retaining the three-dimensional nature of both a wing geometry and its flight path. In the general framework of inviscid incompressible irrotational fluid, thin wakes are represented by vortex sheets. Two broad types of methods currently exist, differing in the way they consider the wake.

The first considers the wake, usually approximated by trapezoidal elements, as a distribution of doublets on its surface. Assuming constant doublet density, this is equivalent to closed vortex rings<sup>1</sup> and corresponds to a vortex lattice, which moves with time according to the local velocity field. Computer developments have allowed direct numerical approach of such three-dimensional non-linearized problems (Albano and Rodden<sup>2</sup>, Djodjodihardjo and Widnall<sup>3</sup>, Katz and Maskew<sup>4</sup>, Mook<sup>5</sup>, Johnson and al.<sup>6</sup>).

The second type of methods considers the wake as thick enough to be represented by volume vortex particles (Rehbach<sup>7</sup>, Morchoisne<sup>8</sup>, Huberson<sup>9</sup>, Winckelmans and Leonard<sup>10</sup>, Buron<sup>11</sup>). These methods are based on a volume discretization of the vortex vector, i.e. the velocity curl. The time variation of the discrete distribution of fluid particles, with each particle supporting this vortex vector, is described with a Lagrangian integro-differential formulation based on the Helmholtz equation.

Actually, both are not really well founded in theory as more or less based on discrete formulations. The vortex lattice approach, often used because of its competitive numerical capabilities, is built on a discrete view of the problem and overlooks the theoretical problem of a continuous wake evolution with time, so that this can lead to numerical problems for large wake distortions. The above vortex particle models allow to develop an accurate deformation scheme, but has the drawback of high-cost numerical algorithms, especially for the Helmholtz equation, using complex regularized schemes<sup>10,11</sup>.

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This work presents a general nonlinear theory for time-dependent problems of lifting and/or propulsive three-dimensional airfoils of variable geometry in arbitrary motion in a potential flow. In many cases, and this will be done in this paper, they can be considered as thin enough to be assumed to be thin surfaces. The airfoil geometry may be arbitrary in the sense that leading, tip and trailing edges may be curved or kinked and the airfoil may have arbitrary camber and twist.

The problem was first formulated in theory<sup>12,13</sup> with no reference to any discrete approach. It mostly relates to the unsteady thin wake model established by Mudry<sup>14</sup>. An exact continuous scheme was developed from this. It particularly focuses on the wake shedding and time deformation, so as to develop an efficient computational tool for the unsteady airloads and wake geometry. The method has been implemented for computation of helicopter rotor wake<sup>15</sup>. For such applications, it is sufficient to consider free vortex sheets emanating only from the trailing edge because rotor blades have a high aspect ratio.

This paper presents new developments of the method for low aspect ratio wings with evolutive vortex sheets emanating from leading, tip and trailing edges.

## 2 DESCRIPTION OF THE METHOD

### 2.1 Background

Consider  $A$ , a thin airfoil in motion in a surrounding fluid, and  $\mathfrak{I}$  the shedding line of its wake  $\Sigma$  (figure 1). In the limiting case of an inviscid flow, the wake, resulting from the union of the two distinct boundary layers issuing from the lower and upper surfaces of the wing, is an infinitely thin layer. It is a slip surface, and, as it cannot sustain any pressure difference, is a discontinuity surface for the tangential velocity component of the fluid particles located on each side of this surface.

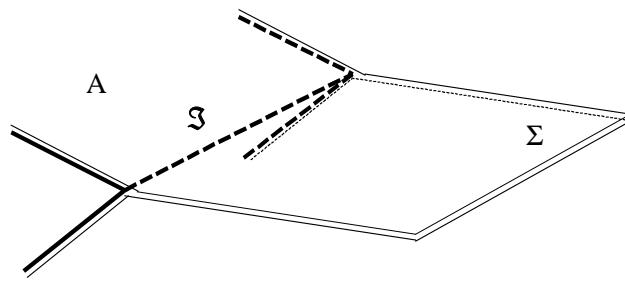


Fig. 1 - Wake in inviscid fluid

The fluid is inviscid and incompressible over the entire irrotational flow field, excluding wing and wake. Therefore, a velocity potential  $\phi$  is defined in an inertial frame of reference, which satisfies Laplace's equation, and is submitted to the following boundary conditions:

- flow tangency (zero normal velocity across the wing),
- quiescent free stream fluid (automatically fulfilled with the below problem formulation),
- smooth flow off the shedding edges (Kutta-Joukowski condition),
- pressure continuity across the wake.

Green's theorem allows to write the velocity potential, on and outside the surfaces  $A$  and  $\Sigma$  :

$$(1) \quad P \in \mathbb{R}^3 \quad \mapsto \quad \phi(P) = -\frac{1}{4\pi} \iint_A \sigma(M) \frac{\vec{r} \cdot \vec{n}_M}{r^3} dS - \frac{1}{4\pi} \iint_{\Sigma} \mu(M) \frac{\vec{r} \cdot \vec{n}_M}{r^3} dS$$

where  $\sigma$  and  $\mu$  are doublet strengths over the surfaces, respectively  $A$  and  $\Sigma$ .  $P$  is a field point,  $\vec{n}_M$  the outward normal in  $M$ , and  $\vec{r} = MP$ ,  $r = \|\vec{r}\|$ . For  $P$  on respectively  $A$  or  $\Sigma$ , the first or second integral in Equation(1) is a Cauchy singular integral, which has to be evaluated in the principal value sense. The derivation of the velocity potential on any surface  $S$  at any point in the field, leads to the resulting formulation (Mudry<sup>14</sup>) :

$$(2) \quad \vec{\text{grad}} \phi = -\frac{1}{4\pi} \iint_S \frac{(\vec{n}_M \wedge \vec{\nabla} \mu(M)) \wedge \vec{r}}{r^3} dS + \frac{1}{4\pi} \int_{\partial S} \mu(M) \vec{\text{grad}}_M \left( \frac{1}{r} \right) \wedge dM$$

in which  $\partial S$  is the oriented edge of  $S$ ,  $\vec{\nabla}$  is the surface gradient of a scalar quantity defined on  $S$  relatively to a local frame and  $\vec{\text{grad}}_M$ , the gradient of a scalar quantity defined in  $\mathbb{R}^3$  and evaluated at a point  $M$  of  $S$ . This expression shows the usual two terms: a surface integral, involving the surface gradient of

the doublet density distribution on the surface, and a curvilinear one involving the value of doublet density distribution along the edge of the surface. Mudry<sup>11</sup> showed that the second term in (2) has to be equal to zero, so that  $\mu$  has to be zero at any free edge of  $S$ , union of the thin airfoil  $A$  and its wake  $\Sigma$ .

## 2.2 Thin wakes and the concept of vortex particles

In the general framework of potential flows, Mudry<sup>16</sup> developed a general theory for unsteady wakes, considered as vortex sheets (discontinuity surface for the tangential velocity) represented by classic double layers. Mudry founded his theory on the median layer concept<sup>17</sup>, which was first introduced by Helmholtz in terms of median velocity. The vortex sheet  $\Sigma$  representing the wake is characterized by the non-unique pair of functions:

-  $\vec{\omega}$ , the "median parametrisation", shown in figure 2, which determines the geometrical shape of the vortex sheet and its deformation :

$$(3) \begin{cases} (q^1, q^2) \in \Omega \subset \mathbb{R}^2 \\ x = (x_1, x_2, x_3) \in \mathbb{R}^3 \end{cases} \quad t \in I \subset \mathbb{R} \mapsto \bar{x} = \vec{\omega}(q^1, q^2, t) \in \Sigma_t$$

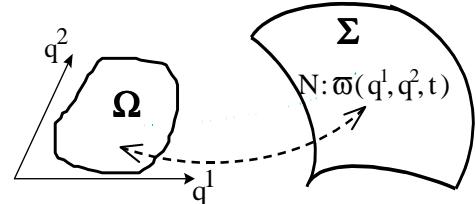


Fig. 2 - Parametrisation of a surface.

-  $\vec{\gamma}$ , the associated "median vortex density" that describes the vortex sheet strength. It is a function of the local velocity jump and of the geometrical description of the wake carried out by the median parametrisation:

$$(4) \quad \vec{\gamma}(q^1, q^2, t) = \vec{N} \wedge [\vec{U}] = \left( \frac{\partial \vec{\omega}}{\partial q^1} \wedge \frac{\partial \vec{\omega}}{\partial q^2} \right) \wedge [\vec{U}] = \gamma^1 \frac{\partial \vec{\omega}}{\partial q^1} + \gamma^2 \frac{\partial \vec{\omega}}{\partial q^2}$$

The wake convection is described by the following evolution equation that  $\vec{\omega}$  verifies :

$$(5) \quad \frac{\partial \vec{\omega}(q^1, q^2, t)}{\partial t} = \frac{\vec{U}^+ + \vec{U}^-}{2} = \vec{U}^*(\vec{\omega}(q^1, q^2, t))$$

where  $\vec{U}^*$  is the median velocity field due to the flow, i.e. the half-sum of the two fluid surfaces velocities.

The position of a vortex particle being defined by the point  $N$  :  $t \mapsto \vec{ON} = \vec{\omega}(q^1, q^2, t)$ ,  $(\vec{\omega}, \vec{\gamma})$  clearly states the concept of continuous vortex particle to represent the wake.

From the choice of the parametrisation  $\vec{\omega}$  together with the pressure continuity condition through the wake, Mudry demonstrated the fundamental time-conservation property for the two contravariant components  $\gamma^\alpha$  ( $\alpha=1,2$ ) of the median vortex density. The wake shedding is taken into account by specifying that  $q^1$  is the shedding edge parameter linked to a shedding edge representation:  $q$ , and  $q^2$  the shedding time:  $\tau$ . It can be deduced that the two contravariant components of the vortex density are determined once for all when the particle is shed with the determined shedding relative velocity at and relatively to  $\mathfrak{I}$  :

$$(6) \quad \vec{V}_{re} = \left( \frac{\partial \vec{\omega}(q, \tau, \tau)}{\partial t} \right)_e - \frac{\partial \vec{h}(q, \tau)}{\partial t}$$

where  $\frac{\partial \vec{h}}{\partial t}$  is the absolute velocity of the shedding edge points by using a lagrangian representation

$\vec{OP} = \vec{h}(q, t)$  of the shedding edge  $\mathfrak{I}$  in the inertial frame.

The velocity induced by the vortex sheet  $\Sigma$  takes the forms :

$$(7) \quad \vec{U}(\bar{x}, t) = \frac{1}{4\pi} \iint_{\Omega} \frac{\vec{\gamma}(q, \tau, t) \wedge (\bar{x} - \vec{\omega}(q, \tau, t))}{\|\bar{x} - \vec{\omega}(q, \tau, t)\|^3} dq d\tau + \frac{1}{4\pi} \int_{\mathfrak{I}} [\Phi](q, \tau, t) \frac{(\bar{x} - \vec{\omega}(q, \tau, t)) \wedge d\vec{\omega}(q, \tau, t)}{\|\bar{x} - \vec{\omega}(q, \tau, t)\|^3}$$

where  $\Phi$  is the velocity potential and  $\mathfrak{I}$  means shedding edge. The integration domain is a known range defined by the definition plane  $(q, \tau)$  of the parametrisation  $\vec{\omega}$ . It does not explicitly depend on time.

The problem of computing the wake, which consists in determining the position and density of the vortex particles once shed, reduces to solving the integro-differential equation that governs the new position of the vortex particles at every moment.

### 2.3 General Nonlinear Equations

The unsteady motion of the airfoil, prescribed in an inertial frame of reference  $R_G = (O ; X, Y, Z)$ , is written in an airfoil-fixed frame  $R = (O' ; x, y, z)$ , as shown in figure 3, and corresponds to a rigid airfoil motion (velocity:  $\vec{V}_E$ ). An optional relative motion within  $R$  describes the deformations of the airfoil, in addition to its average motion.

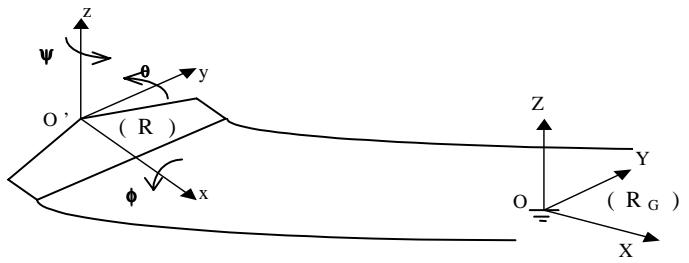


Fig. 3 - Airfoil-fixed frame  $R$  and galilean frame  $R_G$ .

Whereas the geometrical shape of the wake  $\Sigma$  is unknown, it is known for the thin airfoil  $A$ . As for the wake, a parametrisation  $\vec{\chi}$  defined in the inertial frame. The flow tangency condition governs the potential discontinuity  $\phi^+ - \phi^- = [\phi]$  on  $A$ .  $[\phi]$  has to be equal to zero on the borders of  $A$  and  $\Sigma$ , excluding their common border  $\mathfrak{I}$ . Through  $\mathfrak{I}$ , its variations will be governed by the Kutta-Joukowsky condition<sup>18</sup>.

Figure 4 shows how the physical problem is then formulated in two domains, respectively, related to the definition plane of the parametrisations  $\vec{\omega}$  and  $\vec{\chi}$ . This is one of the original features of this approach, as the problem is formulated and solved for both vortex sheets (airfoil and wake) in their respective two-dimensional parametrisation planes.

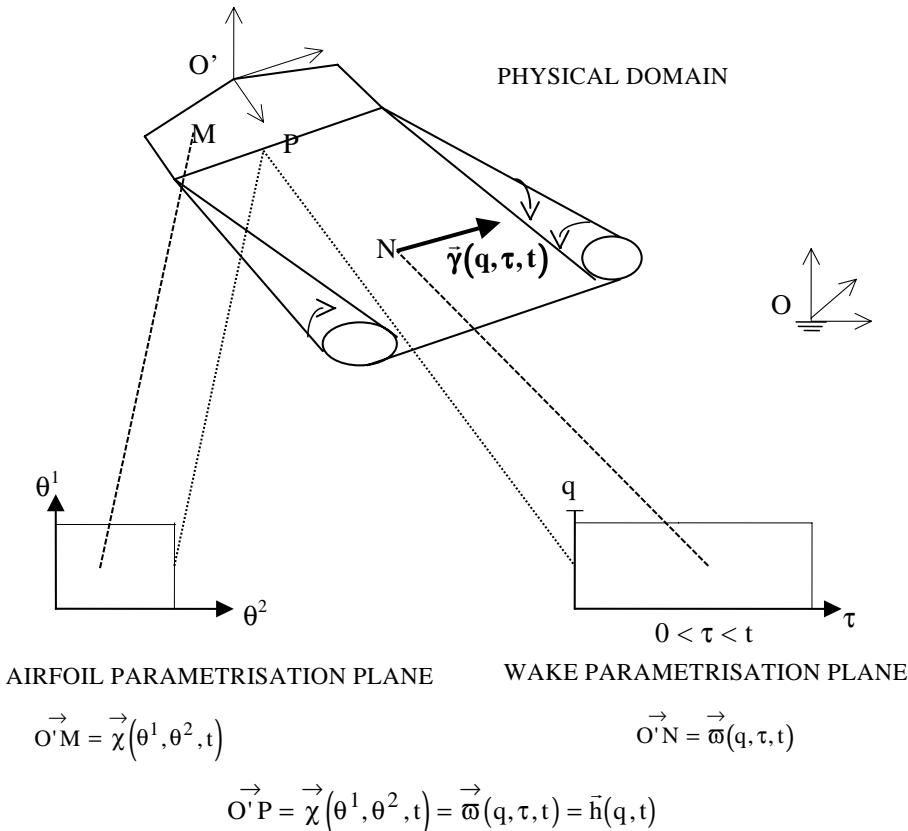


Fig. 4 - Physical field and parametrisation planes.

The nonlinear theoretical problem of a thin airfoil in an unsteady flow is settled. Flow tangency, shedding relative velocity, Kutta-Joukowsky and wake time-variation equations form a nonlinear integro-differential system to be solved which solution requires a numerical approach.

In many usual configurations, the wake shedding at a trailing edge can satisfactorily be prescribed on the shedding edge flight path. In this case,  $\vec{V}_{re}$  is assumed to be equal to the flight path velocity of  $\mathfrak{I}$   $-\frac{\partial \vec{h}}{\partial t}$ , as it is predominant. Thus, the integro-differential system solution is simplified since  $\vec{V}_{re}$  is no longer an unknown. On the other hand, when considering high angles of attack and thus separated flows with wake shedding from airfoil tips or leading edge, this assumption is no longer valid. Indeed, in this case, the shedding relative velocity  $\vec{V}_{re}$  has to consider both terms, the resulting velocity induced by the airfoil and its wake  $\left(\frac{\partial \vec{\omega}}{\partial t}\right)_e$ , as well as the kinematic velocity  $-\frac{\partial \vec{h}}{\partial t}$ . In these conditions  $\vec{V}_{re}$  is an unknown for the shedding process, which will have to be included in an iterative computing process.

The pressure jump can be computed by the Bernoulli equation in the airfoil-fixed system of reference. The pressure difference across the airfoil is then:

$$(8) \quad \frac{\Delta p}{\rho} = (\vec{U}^* + \vec{V}_E) \cdot \vec{V}[\phi] + \frac{\partial [\phi]}{\partial t}$$

The pressure and the potential field, allow to compute the normal, lift and drag forces and corresponding moments, as well as all the desired flow characteristics, such as surface velocity surveys.

## 2.4 Discrete formulation and numerical aspects

Discretisation of the two geometrical surfaces,  $A$  and  $\Sigma$ , is performed through approximations of  $\vec{\chi}$  and  $\vec{\omega}$  defined in relation with a subdivision of their respective definition planes. The simplest possibility is to approximate the definition planes by the union of small uniform quadrilateral panels and to approximate  $\vec{\chi}$  and  $\vec{\omega}$  by their values at the center of these panels. These approximate values of  $\vec{\chi}$  and  $\vec{\omega}$  are then the control or collocation points, where discrete equations are solved.

A step distribution of the doublets over the panels has been retained for the airfoil. This distribution is equivalent to a vortex ring distribution in the definition planes of  $\vec{\chi}$ . Consequently, this type of approximated distribution is not very different from a classical vortex lattice approach, except that the discretisation is performed in the parametrisation planes.

The availability of the continuous theory allowed Mudry to introduce for the wake a discrete vortex particle concept deriving directly from the its continuous formulation, by considering a step distribution of the contravariant components  $\gamma^\alpha$  ( $\alpha=1,2$ ) of the vortex density over the panels. Using  $i,j$  as indexing letters referring to a wake panel, the velocity induced by a discrete vortex particle distribution is then :

$$(9) \quad \vec{U}(x) \approx \sum_{ij} h^1 h^2 \left\{ \gamma_{ij}^q \frac{\left(\frac{\partial \vec{\omega}}{\partial q}\right)_{ij} \wedge (\vec{x} - \vec{\omega}_{ij})}{\|\vec{x} - \vec{\omega}_{ij}\|^3} + \gamma_{ij}^\tau \frac{\left(\frac{\partial \vec{\omega}}{\partial \tau}\right)_{ij} \wedge (\vec{x} - \vec{\omega}_{ij})}{\|\vec{x} - \vec{\omega}_{ij}\|^3} \right\}$$

where  $h^1$  and  $h^2$  are increments of the discrete plane  $(q,\tau)$ . In equation 9, the  $\vec{\omega}$ -derivatives referred to the definition plane have to be approximated, using a finite-difference scheme.

It is well known that any discretisation of this type (vortex ring, discrete vortex particle) introduces a fluid velocity singularity, whereas the continuous formulation does not. This is the case as  $\vec{x}$  tends towards the vortex particle point, i.e. when  $r$  tends towards zero. In order to avoid numerical divergence and thereby regularize the singular behavior of the discrete velocity field, a regularization process is required. According to reference 12, what has been retained is multiplying the induced velocity by the

coefficient  $\frac{r^2}{r^2 + rc^2}$ , where  $rc$  is known as the regularization radius. In many study tests, this radius has been chosen equal to one hundredth of the mean chord.

### Numerical Method

The classical numerical scheme is based on a time-stepping procedure. The time interval  $[0, T]$  is divided into intervals  $\Delta t$ . At initial instant ( $t = 0$ ) no wake exists. At each time step the airfoil is moved along its flight path and the corresponding shed wake can be predicted from the set of discrete equations. At each time step, the wake strip shed during the current time step is a row of free vortex ring elements. Their positions have to be calculated by the shedding process, which is governed by two coupled equations, the Kutta-Joukowsky condition and the time variation equation. Each panel is associated with a vortex ring or a vortex particle by changing the discretisation model.

At each control point on the airfoil, the local velocity induced by the airfoil and its wake, has to satisfy the flow tangency condition. At this stage, the unknowns are the strengths of the airfoil vortex rings and the characteristics of the vortex rings currently shed. The position of each of the other wake discrete vortex particles has been determined by solving the wake time variation equation during the previous time step, and its strength by the Kutta-Joukowsky condition, once for all when shed.

In the case of a trailing edge wake shedding, the positions of these vortex rings elements are derived directly from the shedding assumption  $\vec{V}_{re} = -\partial \vec{h} / \partial t$ . Thus, in the physical domain, the shed vortex ring elements correspond to the interval covered by the trailing edge during the time step. Concerning their strengths, they are determined by applying the Kutta-Joukowsky condition. In this case, the discrete Kutta-Joukowsky condition<sup>18</sup> then consists in setting the strengths of the vortex ring elements equal on both sides (airfoil and wake) of the trailing edge. Using the wake shedding procedure, the discretisation of the flow tangency equation derived at the control points yields to a linear system of algebraic equations, where the unknowns are the strengths of the airfoil vortex rings, and which take into account the strengths of the currently shed vortex rings because of the Kutta-Joukowsky condition. Further details on the procedure may be found in reference 12 and 13. At each time step, the solution is obtained by inverting the matrix of the linear system. If the shape of the airfoil remains unchanged, then the matrix inversion occurs only once.

When considering airfoil tips or leading edge shedding, each vortex ring element of the wake strip is shed from the separation line according to a local shedding relative velocity (Eq. 6). This velocity depends on the strengths of the vortex ring elements currently being shed, which are unknown, so an iterative numerical process has to be used. At each iteration, the shedding relative velocity is obtained from the strengths of the vortex rings in process of shedding and computed at the previous iteration. It is computed on the control point of the neighboring airfoil panels, where a slip condition is verified, so that each vortex element is shed in a local plane tangent to the airfoil at  $\mathfrak{I}$ . The Kutta-Joukowsky condition and the flow tangency equation, which have now to be applied at each iteration, are processed the same way as above.

The time variation equation is discretized on the same basis and its numerical solution actually gives the new position of the vortex particles at each time step. This solution is obtained by using a Runge-Kutta algorithm. The contravariant components of  $\vec{\gamma}$  are computed from the approximation of  $[\phi]$ , which has been determined once for all at the shedding time by the Kutta-Joukowsky condition. The resolution order of the RK-algorithm depends both on computation time and accuracy to predict the wake geometry. The higher the order, the better the wake rollup is simulated.

One part of this work was to carry out some numerical experiments on simple configurations in order to test the convergence of the numerical process in relation with the discretisation parameters, such as the time-step and the number of airfoil panels chordwise and spanwise. The results have shown that the different numerical schemes behave quite well. In every case, convergence was obtained by improving the discretisation refinement (increasing the number of airfoil panels chordwise and spanwise and decreasing the time-step) as shown in references 12 and 13.

### 3 RESULTS

#### 3.1 Application to Flapping Variable-geometry Wings

To illustrate the capabilities of the method, we present the calculation of flapping wings of variable geometry, based on works about the forward flight of birds<sup>16,17</sup>. Consider a pair of flapping wings, which planform shape is a good approximation of a pigeon's, flying with a constant forward velocity  $\bar{U}_0$  and hinged about a longitudinal axis x (figure 5).

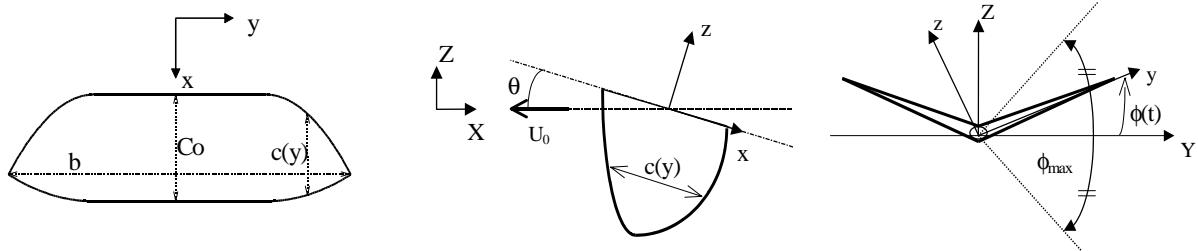


Fig. 5 - Flapping wing planform - Flapping plane - Flapping angle.

One complete period of the flapping motion consists of a downstroke, an upstroke and two transient motions. The upstroke and the downstroke are chosen equal. According to references 15 and 16, the downstroke produces lift and thrust, whereas lift and drag are at their smallest during the upstroke. The assumption of constant angular velocity of flapping  $\dot{\phi}$  is applied during the upstroke and the downstroke. During the transient motions,  $\dot{\phi}$  has a sinusoidal variation. A wing twist allows to prevent unrealistic geometrical incidences resulting from the combination of the forward and flapping motions. This twist is such as the resulting geometrical incidence does not exceed a set incidence  $\alpha_{\max}$  during the downstroke, and is zero during the upstroke. Numerical tests showed that the best results were obtained with a rectilinear twist rotation axis set at the three-quarter line chord. The following results were obtained with the following parameter values :

$$\text{reduced flapping period : } \tilde{T} = \frac{10c_0}{U_0}, \quad \theta = 0^\circ, \quad \phi_{\max} = 80^\circ, \quad \alpha_{\max} = 20^\circ$$

Due to its symmetry, only half of the problem is considered. Discretisation for the wake geometry computation, involved 4 equally spaced chordwise elements and 30 spanwise with a non-dimensional time-step  $\frac{U_0 \Delta t}{c_0} = 0.5$ . The airload computation requires a more refined one : 10 x 40 elements, with

$\frac{U_0 \Delta t}{c_0} = 0.2$ . For 2.5 periods, and without any particular optimization, the computation run-time values

range respectively from some minutes to a few hours on a personal computer. The numerical results show the desired characteristics, i.e. lift and thrust are essentially produced on the downstroke, whereas the lift and drag are negligible on the upstroke as shown in figure 6.

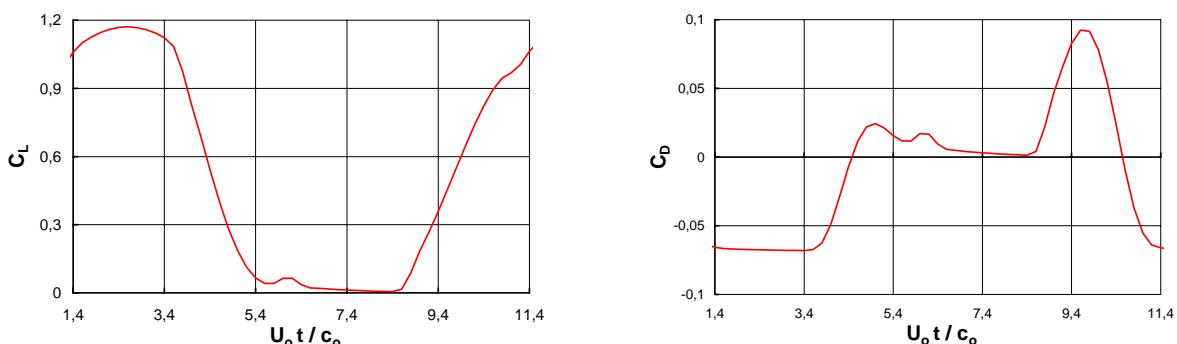


Fig. 6 – Lift and drag/thrust coefficient time variation.

Figure 7 displays the wake geometrical shape at the reduced time  $\frac{U_o t}{c_o} = 20$  after 2.5 calculated periods.

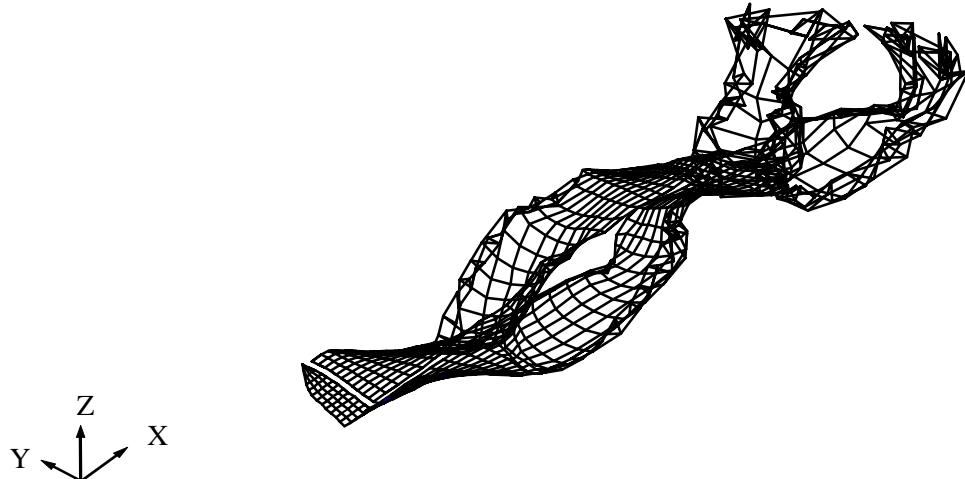


Fig. 7 - Wake geometry at  $\frac{U_o t}{c_o} = 20$ .

### 3.2 Low aspect ratio rectangular wing

Here are presented results of computation including tip wake shedding. It concerns the flow around a low aspect ratio (AR) flat plate at moderate angles of attack.

As a first test case, the sudden acceleration of a wing that was initially at rest is investigated. Computed results for lift coefficient and spanwise lift distribution are shown in figures 8 and 9. They are compared with experimental steady measurements obtained by Scholz in reference 21 for an AR = 2 wing.

Values are obtained for a reduced time of  $\frac{U_o t}{c} = 5$ , which corresponds to five-covered mean chord  $c$ .

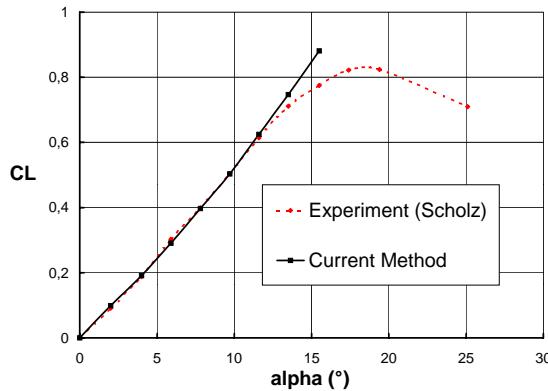


Fig. 8 - Lift coefficient.

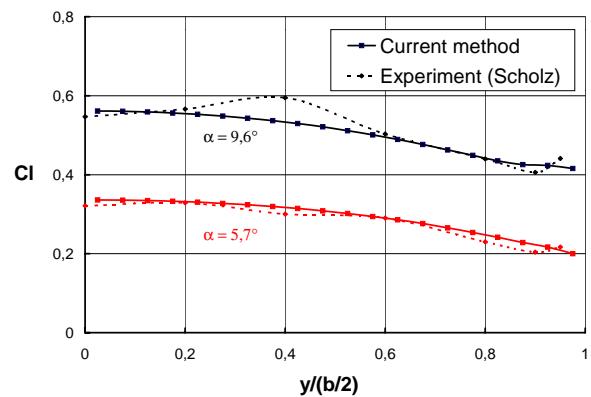


Fig. 9 - Spanwise lift distribution.

The wing is divided into 10x20 elements, chordwise and spanwise. The time step is  $\frac{U_\infty \Delta t}{c} = 0.1$ . Computed results with the current method compares well with experimental values. In order to predict loads for higher angles of attack, it would be necessary to implement a leading-edge-emanating wake. Such model is being implemented.

The second test case presented in figure 10 concerns a unit aspect ratio wing. The wing is divided into 15x15 elements, chordwise and spanwise. The time step is  $\frac{U_\infty \Delta t}{c} = 0.06$ . The unsteady motion is defined by sinusoidal oscillations of the freestream direction with time, from incidence  $0^\circ$  to  $\pm 14.5^\circ$ . Non-dimensional period of oscillations corresponds to one covered chord. Incidence, normal load coefficient

CN and “steady “ normal load CNS coefficient are represented during two periods on figure 8. CNS is equal to CN without the unsteady contribution of the term  $\frac{\partial \phi}{\partial t}$ .

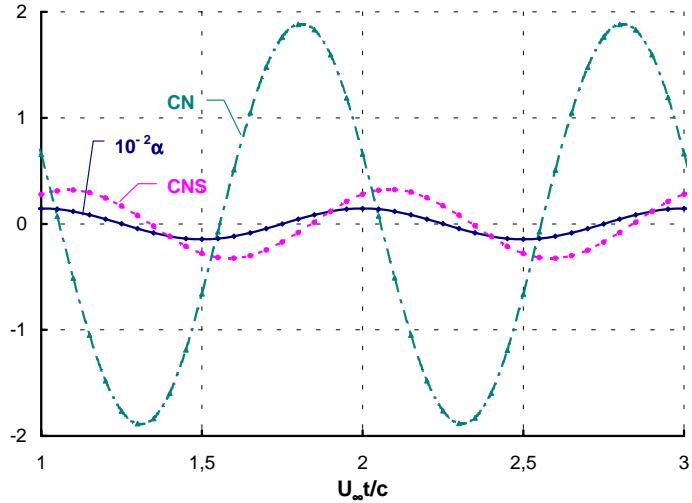


Fig. 10 – Normal force coefficient. Sinusoidal oscillation of incident freestream ( $\pm 14.5^\circ$ ).

It can be noted that the unsteady effect of forces, proportional to  $\frac{\partial \phi}{\partial t}$ , leads to a lead phase. CN higher values are obtained before incidence corresponding values. These computed results compares well with results obtained by Rehbach in a similar test case exposed in reference 7.

A pitching oscillation motion of the wing is also computed. Pitch oscillations from  $0^\circ$  to  $\pm 11.5^\circ$  are about the leading edge and the non-dimensional period of oscillations corresponds to 2.5-covered chord. The difference with the above motion is in the fact that pitching angular velocity,  $\dot{\theta}$ , is taken account in the motion description in the wing-fixed frame defined in section 2.3. It contributes to emphasize unsteady effects on loads and wake geometry. Figure 11 shows time variations of pitch, lift coefficient and pitch moment coefficient.

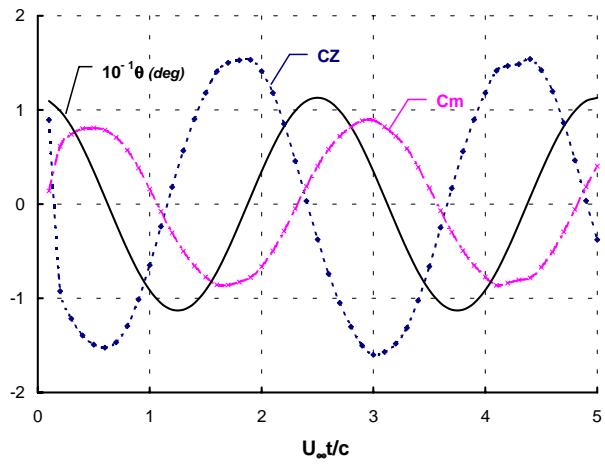


Fig. 11 – Pitching oscillations: lift and pitch moment coefficient.

Figure 12 displays the wake geometry obtained at the reduced time step  $\frac{U_\infty t}{c} = 3.5$ . Desired tip roll-ups are clearly noted and stability of wake simulation is demonstrated.

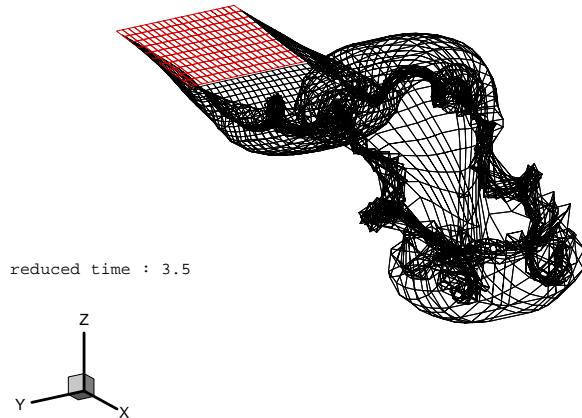


Fig. 12 - Wake geometry at  $\tilde{t} = \frac{U_\infty t}{c} = 3.5$ .

### 3.3 Delta wings

Since most fighter aircraft incorporate a delta-wing surface, there is great interest in studying unsteady delta-wing manoeuvres including pitching to large incidence angle and high-rate rolling motion. Simulation of flow separation from delta wings is being implemented. In order to predict the high-speed flow induced by leading-edge vortices, the current method in its state, and especially for delta wings, needs to include a more refined shedding process, which is being tested.

First results are presented in figure 13 for an unit aspect ratio delta wing with an unit mean chord. This is obtained for an impulsive start at 25 degrees. Figure 13 displays the wake geometry obtained at the reduced time step  $\frac{U_\infty t}{c} = 4$ . The reduced time step is  $\frac{U_\infty \Delta t}{c} = 0.04$  and the wing model consists of 80 panels with 8 chordwise equally spaced panels.

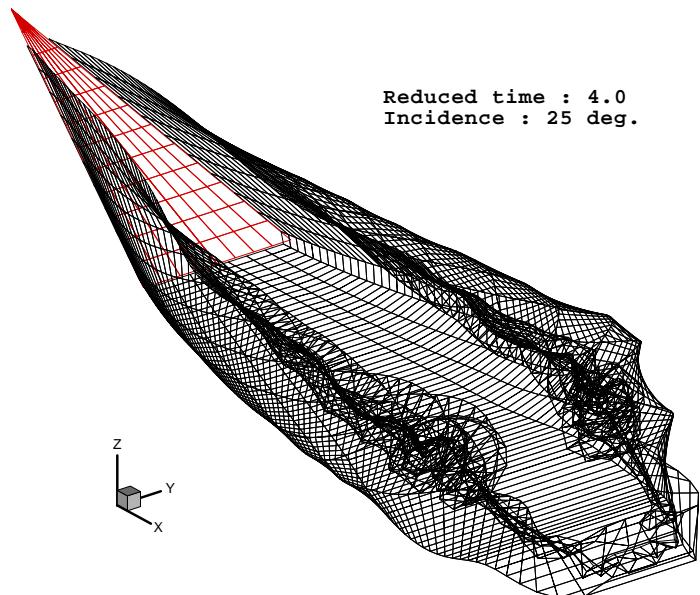


Fig 13 - Wake geometry at  $\tilde{t} = \frac{U_\infty t}{c} = 4$ .

## 4 CONCLUSIONS

On a basis of a rigorous and complete continuous theoretical formulation, a numerical method is developed to solve the unsteady 3D aerodynamics for thin lifting system. Applications of the current method exposed in this paper demonstrates that the theoretical model exposed in section 2 can extend to separation lines, along which the vortex sheets are shed, such as wing tips or parts of the leading edge.

The theory leads to a view of the vortex wake similar to that of a classical vortex lattice approach, but uses a discrete vortex particle concept, which is particularly well suited for computation of the unsteady deforming wake. In fact, the present method allows a treatment of wake deformations comparable with the vortex particle methods quoted as references, though without their limitations, and compares with classical vortex lattice approaches in terms of computing costs. For all results presented in this paper, the computation run-time doesn't exceed one hour on a computer with a Pentium II 350Mhz processor.

The method in its current state is based on the assumption that the flow is symmetrical. In order to simulate more complex and no symmetrical flight configurations, first future developments need to free this assumption and also to consider multi-element systems.

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**PAPER: 10**  
**AUTHOR: DR. LEROY**

**Question by Dr. Hummel:** Have you applied your method to a flow situation for which it is known from experiment that vortex breakdown occurs?

Answer: Not all flow situations are covered.

**Question by Dr. Nangia:** About numerical stability – the close approach problem of 2 vortex lines or more - Do you assume cut-off distances to stabilize the solutions?

Answer: We assume cut-off distances, which directly derive from the discretization models of such methods. For the current method, we explained, in our corresponding paper, that these distances have been chosen to be equal to one hundredth of the mean chord in many study tests.

**Question 2:** Fidelity of vortex flows and roll-ups – in 60's and 70's this method was pioneered. The main difficulty was that rolls-ups in close detail were not accurate enough, e.g. for delta wings. This is presumably due to discretization distances. How fine does one need to go?

Answer: Because of computer capabilities, we have now the possibility of improving the discretization refinement (increasing the number of airfoil panels chordwise and spanwise and decreasing the time-step) in order to obtain "convergence" of the discretization model in relation with the discretization parameters. Some examples are shown in our paper and some quoted among the references. But in complex configurations, e.g. delta wings, in order to predict the high-speed flow induced by leading-edge vortices, it is also necessary to implement the fine shedding process, particularly for vortex sheets emanating from leading edges, and to increase the resolution order of the Runge-Kutta algorithm as well. The higher the order, the better the wake rollup is simulated. This is what we are testing for delta wing computations with the current method.

**Question by Dr. Lamar:** Are there comparisons of this method with other methods, including that of Professor Eddie. Lan, University of Kansas, published in 1981?

Answer: We present some comparisons with other methods (vortex lattice and vortex particle classical methods) in our paper and in references 12 and 13, but we have no comparison with Professor Lan's method because we do not know yet of this reference.

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